Rational Speech Act models are utterance-independent updates of world priors

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Introduction to RSA

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

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The literal listener L_0

$$P_{L_0}(w|u) \propto \mathbb{1}(w \geq n) \times P(w)$$

w	5	6	7
'JP ate 5 cookies'	1/3	1/3	1/3
'JP ate 6 cookies'	0	1/2	1/2
'JP ate 7 cookies'	0	0	1

The pragmatic speaker S_1

$$P_{S_1}(u \mid w) \propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}}$$

(Assume $\alpha = 4$, and that $C(u)$ is constant.)

w	5	6	7
'JP ate 5 cookies'	1	0.16	0.01
'JP ate 6 cookies'	0	0.84	0.06
'JP ate 7 cookies'	0	0	0.93

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w	5	6	7
‘JP ate 5 cookies’	0.85	0.14	0.01
‘JP ate 6 cookies’	0	0.93	0.07
‘JP ate 7 cookies’	0	0	1

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- Discuss some consequences:
    - for computing RSA models;
    - for the algorithmic plausibility of RSA (Marr, 1982).

RSA via information gain

Information gain as K-L divergence

Kullback-Leibler (K-L) divergence (between distributions P and Q over $\mathcal{X}$):

$$D_{\mathsf{KL}}(Q \parallel P) = -\sum_{x \in \mathcal{X}} Q(x) \log\left(\frac{P(x)}{Q(x)}\right)$$

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Higher values: more information is gained by going from P to Q.

A theorem

Theorem
If $Q(x) \propto f(x) \times P(x)$, and the range of f is $\{0, 1\}$, then:

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In words: if f updates P merely by *filtering* it (resulting in Q), then the information gain of Q is the negative log of the expected value of f.

- Important: the expected value of f is *also* the normalizing constant for Q:

$$Q(x) = \frac{f(x) \times P(x)}{\sum_{x'} f(x') \times P(x')}$$

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u, w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u, w') \times P(w')}$$

where $l(u, w) = 1$ or 0, depending on whether u is true at w

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Let us define the *literal information gain* of u:

$$G_{L_0}(u) = D_{\mathrm{KL}}(P_{L_0}(w|u) \parallel P(w))$$

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$$P_{L_0}(w|u) = l(u,w) \times P(w) \times e^{G_{L_0}(u)}$$

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The pragmatic speaker S_1:

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Upshot:

- $l(u,w)$ is our filter
- $P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$ is our *new* "prior"!

Reformulating the pragmatic speaker: example

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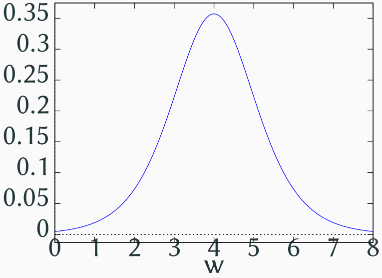
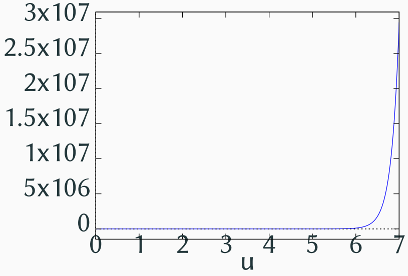


Prior over JP's running distance



$P_{S_1}(u|w = 7)$ with $\alpha = 4$

Making the normalizing constant explicit

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$$G_{S_1}(w) = -\log \sum_{u \in \mathcal{U}} l(u,w) \times P_{S_1}(u)$$

the *specificity* of w

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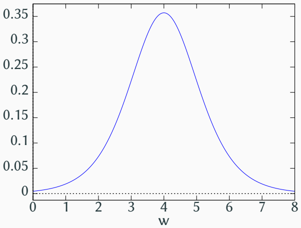

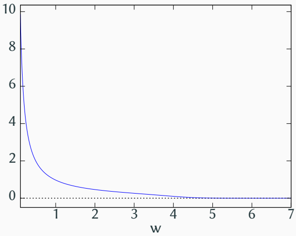

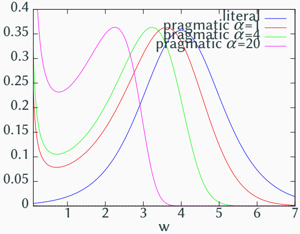

$$P(w) \quad \times \quad e^{G_{S_1}(w)} \quad = \quad P_{L_1}(w)$$

(JP's distance prior) (specificity) (L_1 "pragmatic" prior)

Some consequences

Computing RSA models

Main result:

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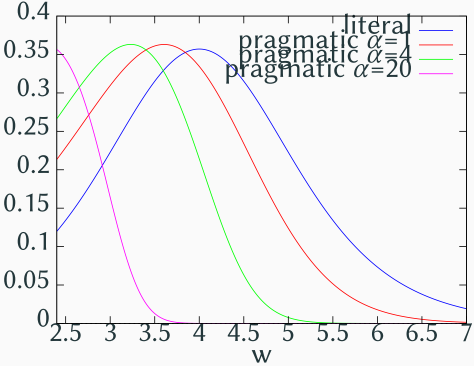

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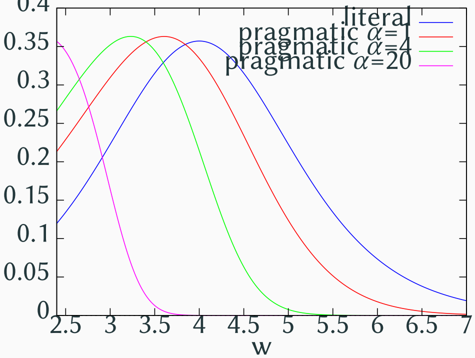

(Side note: the expected implicature is generated for large α only!)

Algorithmic plausibility

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(RSA-style) pragmatic interpretations can be *learned*, just as probability distributions over events can; pragmatic distributions are additionally sensitive to an event's specificity.

References

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Marr, David. 1982. *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information.* Cambridge: MIT Press.