Probabilistic compositional semantics, purely

Julian Grove and Jean-Philippe Bernardy

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CLASP, University of Gothenburg

Motivation

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…generally, by dropping typed λ-calculus and encoding meanings in terms of probabilistic programming languages.

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Such programming languages are often *impure*: they allow for probabilistic effects, like sampling and marginalization, to occur at any point in a program.

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End up with a characterization of meanings as *probabilistic programs*, which are, nevertheless, pure (i.e., no real probabilistic effects).

Such programs *describe* probability distributions over logical meanings.

Schematically...

$$[\![\textit{someone is tall}]\!] \; = \; \exists x : \text{human}(x) \land \text{height}(x) \geq \theta_{tall}$$

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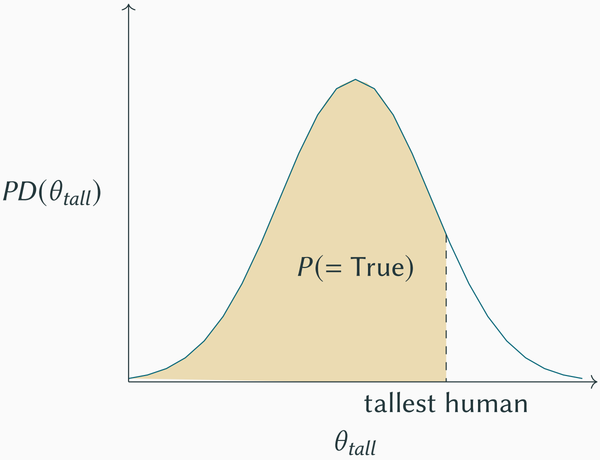
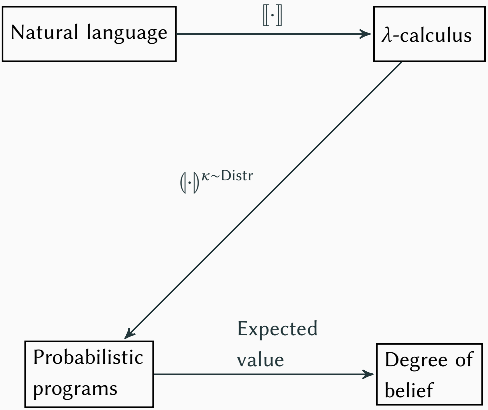
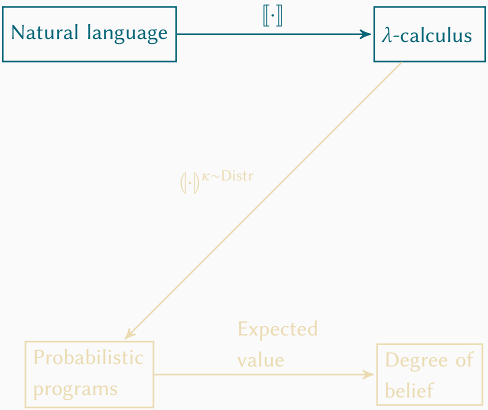


Our system


Natural language
⟦·⟧
λ-calculus
⦇·⦈κ∼Distr
Probabilistic programs
Expected value
Degree of belief

Up next



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Formal semantics

Logics and sets

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Two strategies to formally interpret natural language, inherited from Montague:

- direct: right into set theory
  - denotations (entities, functions, etc.) are elements of sets
- indirect: into a formal logic, e.g., the simply-typed λ-calculus/higher-order logic

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(1) Someone is tall.

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[\![\textit{someone}]\!] &= \lambda k.\exists x : \text{human}(x) \wedge k(x) \\
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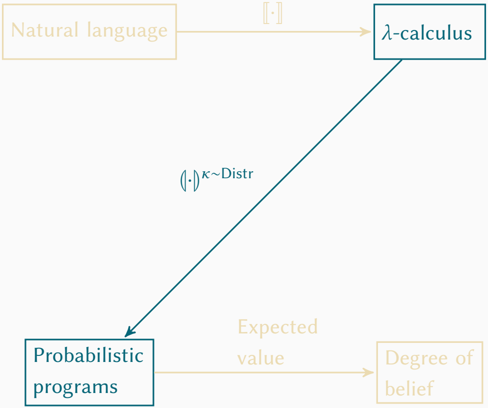
Up next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The probabilistic interpretation

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- A *context* (κ) is a tuple of type $\alpha_1 \times \ldots \times \alpha_n$, where α_i is the type of the i^{th} constant.
- A context for this language would be of type $(e \rightarrow d_{tall}) \times (e \rightarrow t) \times (r \rightarrow r \rightarrow t) \times d_{tall}$.

A λ-homomorphism in a context

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$$(\!|x|\!)^\kappa = x \qquad \text{(variables)}$$

$$(\!|\lambda x.M|\!)^\kappa = \lambda x.(\!|M|\!)^\kappa \qquad \text{(abstractions)}$$

$$(\!|MN|\!)^\kappa = (\!|M|\!)^\kappa (\!|N|\!)^\kappa \qquad \text{(applications)}$$

$$(\!|\langle M, N\rangle|\!)^\kappa = \langle (\!|M|\!)^\kappa, (\!|N|\!)^\kappa \rangle \qquad \text{(pairing)}$$

$$(\!|M_i|\!)^\kappa = (\!|M|\!)^\kappa_i \qquad \text{(projection)}$$

Etc. ($\diamond$, logical constants)

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Goal: allow the context to be a random variable.

Probabilistic programs

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- The probabilistic program that returns Jean-Philippe with a probability of 1.

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“Run m, computing x. Then feed x to k."

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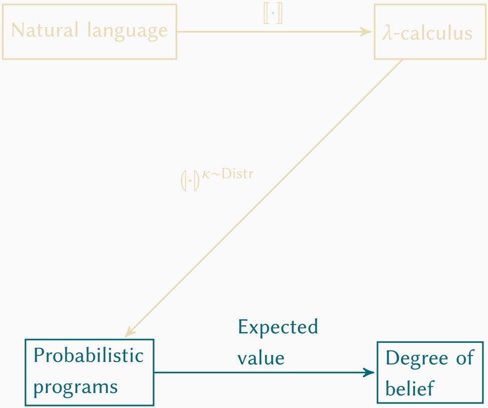
- Then, for a sentence ϕ in the logical language, we may do:

$$K \star \lambda\kappa.\eta((\!|\phi|\!)^{\kappa}) : (t \rightarrow r) \rightarrow r$$

Up next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- $\lambda b.1$ picks out the total mass (assigned to $\top$ and $\bot$).
- So, $P(p)$ is the probability that p returns $\top$.

An example

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Say our constants are:

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Define K as:

$$K = \mathcal{N}(72, 3) \star \lambda d.\eta(height, human, (\geq), d)$$

An example (cont'd)

$$K \star \lambda\kappa.\eta(\llparenthesis \exists x : \text{human}(x) \wedge \text{height}(x) \geq \theta_{tall} \rrparenthesis^{\kappa})$$

An example (cont'd)

$$
\begin{aligned}
& K \star \lambda\kappa.\eta(\llparenthesis \exists x : \mathrm{human}(x) \wedge \mathrm{height}(x) \geq \theta_{tall} \rrparenthesis^{\kappa}) \\
& \vdots \\
= \; & \lambda f.\, \mathcal{N}(72, 3)(\lambda d. f(\exists x : human(x) \wedge height(x) \geq d)) \\
= \; & \lambda f.\, \int_{-\infty}^{\infty} \mathrm{PDF}_{\mathcal{N}(72,3)}(y) * f(\exists x : human(x) \wedge height(x) \geq y) dy
\end{aligned}
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…the mass of $\mathcal{N}(72, 3)$ less than or equal to the height of the tallest human

Bayesian inference (e.g., RSA)

RSA models: a popular application of probabilistic semantics.

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- …who infers a distribution over meanings m from an utterance u, based on the probability that a pragmatic speaker, S_1, would make the utterance u to convey m.

RSA models: a popular application of probabilistic semantics.
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- Given a meaning m, the probability that S_1 would make the utterance u to convey m is related to the probability that a literal listener, L_0, would infer m, given a literal interpretation of u.

Factoring by a weight / observing a premise

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\begin{aligned}
\mathit{factor} &: r \to (\diamond \to r) \to r \\
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RSA: implementation

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Conclusion

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…using the same logical language one uses to characterize linguistic meanings.

References

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