Some questions about vagueness and metalinguistic uncertainty

Julian Grove

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FACTS.lab, University of Rochester

Vagueness versus metalinguistic uncertainty

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C. Therefore, a free cup of coffee is expensive.

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Sorites-like imprecision for uncertainty

However, Lassiter (2011) argues that uncertain factual knowledge can display sorites-like behavior:

> *'There is no real number r such that my belief state allows for the possibility that Big Ben and the Eiffel Tower are r kilometers apart, but excludes the possibility that they are $r \pm \epsilon$ kilometers apart for sufficiently small ϵ.'*

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Still not accessible to sorites arguments…

P2. If the Big Ben and Eiffel Tower are *r* km apart, then they are also 1 mm less then *r* km apart. ✗

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In contrast, uncertain knowledge can be made certain:

(6) A .93-mile road is 1 metric mile, but a .92-mile road is not 1 metric mile.

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In both cases *d* is held constant for the purpose of supporting the entailment from P1 and P2 to C.

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Row 3 suggests that they can be held fixed in certain cases.

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The plan: characterize both vagueness and metalinguistic uncertainty as outcomes of semantic knowledge being probabilistic in nature (Lassiter 2011; Lassiter and Goodman 2013, 2017, i.a.).

- in a pure logical setting, where probabilistic semantic knowledge gives rise to an *applicative functor*
- and by relying on the composition of applicative functors in order to get a handle on the semantic separation between vagueness and uncertainty

Probabilistic semantics via probabilistic programs

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  - Result: the weighted average (i.e., *expected value*) of $f(x)$ across the normally distributed values x.

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- a method of turning ordinary logical meanings into probabilistic programs
- a method of composing probabilistic programs together, similar to how we compose ordinary natural language meanings by functional application

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Viewed this way, the map P is what is known as an *applicative functor*. This means two things…

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(The probabilistic program that returns the coffee in Rome with a probability of 1.)

Applicative functors allow you to compose programs together

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“Run m to compute x. Then run n to compute y. Then apply x to y.”

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- In the above, it picks out the mass assigned to $\top$.
- $m(\lambda b.1)$ is the *measure* of m: it is m's *total mass*.
- So, $Pr(m)$ is the probability that m returns $\top$.

Probabilistic semantics for vagueness

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$[\![(1)]\!] : \mathsf{P}(t)$
$[\![(1)]\!] = [\![\text{expensive}]\!] \circledast [\![\text{the coffee in Rome}]\!]$
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$[\![\text{the coffee in Rome}]\!] = \eta(\text{coffeeInRome})$

$[\![\text{expensive}]\!] : \mathsf{P}(e \longrightarrow t)$
$[\![\text{expensive}]\!] = \lambda f.\mathcal{N}(\mu_{exp}, \sigma_{exp})(\lambda d.f(\lambda x.\text{cost}(x) \geq d))$

$[\![(1)]\!] : \mathsf{P}(t)$
$[\![(1)]\!] = [\![\text{expensive}]\!] \circledast [\![\text{the coffee in Rome}]\!]$
$= \lambda f.\mathcal{N}(\mu_{exp}, \sigma_{exp})(\lambda d.f(\text{cost}(\text{coffeeInRome}) \geq d))$

If, for example, cost(coffeeInRome) $= \mu_{exp}$, then $Pr([\![(1)]\!]) = 0.5$.

(5) If an expensive cup of coffee were 1 cent cheaper, it would still be expensive.

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We need a meaning for *if* ‼

Factoring by a weight / observing a premise

$$\mathit{factor} : r \longrightarrow \mathsf{P}(\diamond)$$

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Sorites (cont'd 1)

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$[\![if]\!](\phi)(\psi)(mb) =$

$$Pr(\lambda f.mb(\lambda w.observe(\phi(w))(\lambda\diamond.f(\psi(w))))) \geq r_{certainty}$$

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"Given some distribution over worlds *mb*, the probability that ψ is true after filtering out the worlds where ϕ is false is greater than some required threshold of certainty $r_{certainty}$."

Sorites (cont'd 2)

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$[\![\textit{an expensive cup were 1 cent cheaper}]\!] : r \times r \rightarrow t$
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Sorites (cont'd 3)

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"If you take the mass of *mb* where $d \geq d' - 0.01$, the proportion of this mass where $d \geq d'$, as well, is greater than the certainty threshold."

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"If you take the mass of *mb* where $d \geq d' - 0.01$, the proportion of this mass where $d \geq d'$, as well, is greater than the certainty threshold."

For example, if d and d' are independently normally distributed with the same mean, this will always be ≥ 0.5. For $\sigma = 1$, it is > 0.99.

Entailments

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"Given a starting discourse c and a proposition ϕ to update it with, $update(c)(\phi)$ is just like c, except that worlds where ϕ is false are assigned a probability of 0."

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$[\![\text{P1}]\!] : r \times r \times r \rightarrow t$
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$[\![\mathrm{P2}]\!] : r \times r \times r \rightarrow t$
$[\![\mathrm{P2}]\!] = \lambda\langle r, g, d\rangle.g > r$

Entailments (cont'd 2)

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$update(update(c)([\![\text{P1}]\!]))([\![\text{P2}]\!]) =$

$$\lambda f.c(\lambda\langle r, g, d\rangle.\mathbb{1}(r \geq d) * \mathbb{1}(g > r) * f(r, g, d))$$

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"The context just like c, except that the only worlds with non-zero probabilities are those where $r \geq d$ and $g > r$ (and, hence, $g > d$)."

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What makes metalinguistic uncertainty different?

Probabilistic semantics for metalinguistic uncertainty

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- Vagueness: $\mathsf{P}(\mathsf{P}(\alpha))$
- Uncertainty: $\mathsf{P}(\mathsf{P}(\alpha))$

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$[\![(2)]\!] : \mathsf{P}(\mathsf{P}(t))$
$[\![(2)]\!] = [\![\text{metric mile long}]\!] \circledast [\![\text{road}]\!]$
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Why no sorites?

The type provided for *if*: $(w \to t) \to (w \to t) \to \mathsf{P}(w) \to t$

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Hypothetical constraint on lexical meanings:

$\mathsf{P}(\mathsf{P}(\alpha))$ can't occur in a negative position in an expression's type.

Entailments?

We need only adjust the type of the *update*, using η:

$$update : \mathsf{P}(\mathsf{P}(w) \rightarrow (w \rightarrow t) \rightarrow \mathsf{P}(w))$$
$$update = \eta(\lambda c, \phi, f.c(\lambda w.observe(\phi(w))(\lambda\diamond.f(w))))$$

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Under the current picture, the phenomena of vagueness arise from two aspects of semantic knowledge conspiring:

- the semantic types of linguistic expressions like *if*
- the encoding of vague probabilistic knowledge on an "inner" applicative layer ($\mathsf{P}(\mathsf{P}(\alpha))$)

This makes room for some probabilistic knowledge not participate in these phenomena — encode them on the "outer" layer ($\mathsf{P}(\mathsf{P}(\alpha))$).

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